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Analysis of the Second Harmonic Effect on Power Amplifier Intermodulation Products

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Abstract—A theoretical analysis is reported in this paper to investigate the effect that a second harmonic signal which might be present at an amplifier's input has on generating additional intermodulation products, particularly the third-order intermodulation (IM3) products. The analysis shows that the amplitude of an extra generated IM3 component is equal to the product of the fundamental amplitude, the second harmonic amplitude, and the second order Taylor series coefficient. The effect of the second order harmonic on the IM3 is examined through a simulated example of a 2.22-GHz 10-W Class-EF amplifier whereby the IM3 levels have been reduced by 2-3 dB after employing a second harmonic termination stub at the input.

Keywords— *Distortion, harmonic termination, IMD secondary mechanism, intermodulation, linearity, power amplifier, two-tone simulation.*

I. INTRODUCTION

Intermodulation is a type of signal distortion which occurs when at least two different frequencies are presented to a non-linear system such as a power amplifier (PA). The analysis and effects of this phenomenon have been reported in literature in many occasions [1]–[7]. Intermodulation can cause two types of distortions, i.e., in-band and out-of-band distortions. The former is a result of changing the amplitude and phase of the fundamental frequency signal, and can increase the bit error rate of the transmitted signal. Whereas the latter, also known as spectral re-growth, presents new frequency components that are formed outside the frequency band of interest. Some of these components, e.g., the even-order intermodulation products and harmonic products are far from the carrier frequency and therefore can be easily filtered out. The other components, i.e., the odd-order intermodulation products lie close to the fundamental signal and manifest in a harmful way that increases adjacent channel interference. The most severe products i.e. due to their significant amplitudes and close proximity to fundamental tones are the third-order intermodulation products (IM3) located at frequencies $(2f_1 - f_2)$ and $(2f_2 - f_1)$ with f_1 and f_2 being the carrier frequencies [1].

Two-tone intermodulation analysis has been discussed in [1] and [8] where two pure sinusoidal signals are introduced to an amplifier's non-linear transfer function. Nevertheless, this is not entirely practical since a number of harmonics might exist at the PA input due to transistor reflections or signal distortions. This paper describes analytically the generation of additional intermodulation products due to the secondary mechanism of the second order harmonics. Section II presents the theoretical

derivation of the intermodulation distortion (IMD) resulting when two fundamental tones and a second order harmonic are present at the PA input. Section III describes a simulated PA circuit showing the effect of the second harmonics on IM3 levels. Finally, research finding reported in paper is summarised in Section IV.

II. ANALYSIS OF THE IMD SECONDARY MECHANISM

A. Idealized Two-Tone Analysis

Two sinusoidal waveforms are presented to an amplifier input as follows

$$V_{IN} = E_1 \sin \omega_1 t + E_2 \sin \omega_2 t, \quad (1)$$

where E_1 and E_2 are the amplitudes of the first and second tones respectively. The non-linear characteristics of the amplifier can be approximately described using a power series expansion. Therefore, the output signal can be written as follows

$$V_{OUT} = K_1 V_{IN} + K_2 V_{IN}^2 + K_3 V_{IN}^3 + \dots \quad (2)$$

where K_1 , K_2 , and K_3 are the power series coefficients. For simplicity, only the first three terms will be considered in the analysis. The first term represents the fundamental components of the output signal, whereas the remaining two terms contain the DC, harmonic and intermodulation components. Thus, for our interest, we will analyze the second and third terms only as follows.

The second term can be expressed as

$$\begin{aligned} K_2 V_{IN}^2 &= K_2 (E_1 \sin \omega_1 t + E_2 \sin \omega_2 t)^2 \\ &= K_2 (E_1^2 \sin^2 \omega_1 t + E_2^2 \sin^2 \omega_2 t \\ &\quad + 2E_1 E_2 \sin \omega_1 t \sin \omega_2 t) \\ &= K_2 \left[\frac{1}{2} (E_1^2 + E_2^2) - \frac{E_1^2}{2} \cos 2\omega_1 t - \frac{E_2^2}{2} \cos 2\omega_2 t \right. \\ &\quad \left. + E_1 E_2 \cos(\omega_1 - \omega_2)t - E_1 E_2 \cos(\omega_1 + \omega_2)t \right]. \end{aligned} \quad (3)$$

Equation (4) shows that the second term in (2) produces DC, even harmonics and even intermodulation products.

The third term can be expressed as

$$K_3 V_{IN}^3 = K_3 (E_1 \sin \omega_1 t + E_2 \sin \omega_2 t)^3. \quad (5)$$

After some mathematical operations, this can be written as

$$K_3 V_{IN}^3 = K_3 \left[\left(\frac{3E_1^3}{4} + \frac{3E_1 E_2^2}{2} \right) \sin \omega_1 t + \left(\frac{3E_2^3}{4} + \frac{3E_1^2 E_2}{2} \right) \sin \omega_2 t - \frac{E_1^3}{4} \sin 3\omega_1 t - \frac{E_2^3}{4} \sin 3\omega_2 t - \frac{3E_1^2 E_2}{4} \sin(2\omega_1 + \omega_2)t + \frac{3E_1 E_2^2}{4} \sin(2\omega_1 - \omega_2)t - \frac{3E_1 E_2^2}{4} \sin(2\omega_2 + \omega_1)t + \frac{3E_1 E_2^2}{4} \sin(2\omega_2 - \omega_1)t \right] \quad (6)$$

As shown in (6), this term produces fundamental, odd harmonics, and odd intermodulation components. The lower and upper IM3 components are

$$IM3_L = \frac{3K_3 E_1^2 E_2}{4} \sin(2\omega_1 - \omega_2)t \quad (7)$$

$$IM3_U = \frac{3K_3 E_1 E_2^2}{4} \sin(2\omega_2 - \omega_1)t \quad (8)$$

B. Two-Tone Analysis with Second Harmonic Input Signal

Now we assume a more realistic case, where a second harmonic component with amplitude H_2 exists alongside the two-tone signal at the input. Assuming the second harmonic component belongs to the lower fundamental frequency, the input signal can be written as

$$V_{IN} = E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + H_2 \sin 2\omega_1 t. \quad (9)$$

Similar to the ideal case in subsection (A), the second term of (2) can be expressed as

$$K_2 V_{IN}^2 = K_2 (E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + H_2 \sin 2\omega_1 t)^2. \quad (10)$$

After some mathematical operations this can be written as

$$K_2 V_{IN}^2 = K_2 \left[\frac{1}{2} (E_1^2 + E_2^2 + H_2^2) + E_1 H_2 \cos \omega_1 t - \frac{E_1^2}{2} \cos 2\omega_1 t - \frac{E_2^2}{2} \cos 2\omega_2 t + E_1 E_2 \cos(\omega_1 - \omega_2)t - E_1 E_2 \cos(\omega_1 + \omega_2)t - E_1 H_2 \cos 3\omega_1 t - E_2 H_2 \cos(2\omega_1 + \omega_2)t + E_2 H_2 \cos(2\omega_1 - \omega_2)t - \frac{H_2^2}{2} \cos 4\omega_1 t \right] \quad (11)$$

Table I Extra output frequency products due to the existence of the second order harmonic component at the system input.

Frequency	Additional generated terms
DC	$\frac{1}{2} K_2 H_2^2$
ω_1	$K_2 E_1 H_2 \cos \omega_1 t + \frac{3K_3 E_1 H_2^2}{2} \sin \omega_1 t$
ω_2	$\frac{3K_3 E_2 H_2^2}{2} \sin \omega_2 t$
$2\omega_1$	$K_3 \left(\frac{3H_2^3}{4} + \frac{3E_1^2 H_2}{2} + \frac{3E_2^2 H_2}{2} \right) \sin 2\omega_1 t$
$\omega_1 + \omega_2$	$\frac{3K_3 E_1 E_2 H_2}{2} \sin(\omega_1 + \omega_2)t$
$\omega_2 - \omega_1$	$\frac{3K_3 E_1 E_2 H_2}{2} \sin(\omega_2 - \omega_1)t$
$3\omega_1$	$\frac{3K_3 H_2^2 E_1}{4} \sin 3\omega_1 t - K_2 E_1 H_2 \cos 3\omega_1 t$
$2\omega_1 + \omega_2$	$-K_2 E_2 H_2 \cos(2\omega_1 + \omega_2)t$
$2\omega_1 - \omega_2$	$K_2 E_2 H_2 \cos(2\omega_1 - \omega_2)t$
$4\omega_1$	$-\frac{3K_3 H_2 E_1^2}{4} \sin 4\omega_1 t - \frac{K_2 H_2^2}{2} \cos 4\omega_1 t$
$2\omega_1 + 2\omega_2$	$-\frac{3K_3 E_2^2 H_2}{4} \sin(2\omega_1 + 2\omega_2)t$
$2\omega_2 - 2\omega_1$	$\frac{3K_3 E_2^2 H_2}{4} \sin(2\omega_2 - 2\omega_1)t$
$3\omega_1 + \omega_2$	$-\frac{3K_3 E_1 E_2 H_2}{2} \sin(3\omega_1 + \omega_2)t$
$3\omega_1 - \omega_2$	$\frac{3K_3 E_1 E_2 H_2}{2} \sin(3\omega_1 - \omega_2)t$
$5\omega_1$	$-\frac{3K_3 E_1 H_2^2}{4} \sin 5\omega_1 t$
$4\omega_1 + \omega_2$	$-\frac{3K_3 E_2 H_2^2}{4} \sin(4\omega_1 + \omega_2)t$
$4\omega_1 - \omega_2$	$\frac{3K_3 E_2 H_2^2}{4} \sin(4\omega_1 - \omega_2)t$
$6\omega_1$	$-\frac{K_3 H_2^3}{4} \sin 6\omega_1 t$

The third term can be expressed as

$$K_3 V_{IN}^3 = K_3 (E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + H_2 \sin 2\omega_1 t)^3. \quad (12)$$

$$K_3 V_{IN}^3 = K_3 \left[\left(\frac{3E_1^3}{4} + \frac{3E_1 E_2^2}{2} + \frac{3E_1 H_2^2}{2} \right) \sin \omega_1 t + \left(\frac{3E_2^3}{4} + \frac{3E_2 E_1^2}{2} + \frac{3E_2 H_2^2}{2} \right) \sin \omega_2 t + \left(\frac{3H_2^3}{4} + \frac{3H_2 E_1^2}{2} + \frac{3E_2^2 H_2}{2} \right) \sin 2\omega_1 t + \frac{3E_1 E_2 H_2}{2} \sin(\omega_1 + \omega_2)t + \frac{3E_1 E_2 H_2}{2} \sin(\omega_2 - \omega_1)t + \left(\frac{3H_2^2 E_1}{4} - \frac{E_1^3}{4} \right) \sin 3\omega_1 t - \frac{E_2^3}{4} \sin 3\omega_2 t - \frac{3E_1^2 E_2}{4} \sin(2\omega_1 + \omega_2)t + \frac{3E_1^2 E_2}{4} \sin(2\omega_1 - \omega_2)t - \frac{3E_1 E_2^2}{4} \sin(2\omega_2 + \omega_1)t + \frac{3E_1 E_2^2}{4} \sin(2\omega_2 - \omega_1)t - \frac{3H_2^2 E_1}{4} \sin 4\omega_1 t - \frac{3E_2^2 H_2}{4} \sin(2\omega_1 + 2\omega_2)t + \frac{3E_2^2 H_2}{4} \sin(2\omega_2 - 2\omega_1)t - \frac{3E_1 E_2 H_2}{2} \sin(3\omega_1 + \omega_2)t + \frac{3E_1 E_2 H_2}{2} \sin(3\omega_1 - \omega_2)t - \frac{3E_1 H_2^2}{4} \sin 5\omega_1 t + \frac{3E_2 H_2^2}{4} \sin 5\omega_2 t + \frac{K_3 H_2^3}{4} \sin 6\omega_1 t \right]$$

$$\begin{aligned} & \frac{3E_1H_2^2}{4}\sin 5\omega_1t - \frac{3E_2H_2^2}{4}\sin (4\omega_1 + \omega_2)t + \\ & \frac{3E_2H_2^2}{4}\sin (4\omega_1 - \omega_2)t - \frac{H_2^3}{4}\sin 6\omega_1t]. \end{aligned} \quad (13)$$

From (4), (6), (11) and (13), the extra products generated due to the existence of the second harmonic are summarized in Table I. Table I shows the additional amount of in-band and out-of-band intermodulation distortions the signal would carry if a second harmonic component is presented at the input of the system. If the 2nd harmonic component belongs to the upper frequency ($2\omega_2$), the results would be the same, with only subscripts “1” and “2” interchanged. The lower and upper IM3 components in this case are

$$IM3_L = \frac{3K_3E_1^2E_2}{4} \sin(2\omega_1 - \omega_2)t + K_2E_2H_2 \cos(2\omega_1 - \omega_2)t, \quad (14)$$

$$IM3_U = \frac{3K_3 E_1 E_2^2}{4} \sin(2\omega_2 - \omega_1)t. \quad (15)$$

From (7), (8), (14) and (15), it can be deduced that a second order harmonic belonging to the lower frequency would generate an extra lower IM3 product, whereas it would not affect the upper IM3, and vice versa, *the extra lower IM3 term having an amplitude of $(K_2E_2H_2)$* . To show the resulting variation occurring to the magnitude and phase of the $IM3_L$, (14) can be written as follows

$$IM3_L = \sqrt{\left[\frac{3K_3E_1^2E_2}{4}\right]^2 + [K_2E_2H_2]^2} \cos((2\omega_1 - \omega_2)t - \tan^{-1} \frac{3K_3E_1^2}{4K_2H_2}). \quad (16)$$

This phase variation might be of importance when intermodulation is to be mitigated through g_{m3} cancellation approach.

III. INTERMODULATION IN CLASS-EF AMPLIFIER

The third-harmonic-peaking Class-EF PA reported in [9] is now investigated for its IM3 behavior. Implemented using a GaN HEMT transistor from Cree (CGH40010F), this PA operates at 2.22 GHz and delivers 40-dBm output power. A two-tone simulation was run for two cases, i.e., with and without 2nd harmonic trap at the input. The circuit schematic of the amplifier is shown in Fig. 1. The circuit is biased with gate-source voltage $V_{GS} = -2.7$ V and drain-source voltage $V_{DS} = 28$ V. The center frequency of the two tones is set to 2.22 GHz with 5 MHz spacing between them, i.e., the lower frequency $f_1 = 2.2175$ GHz and the upper frequency $f_2 = 2.2225$ GHz. The input power is divided equally over the two tones. Input matching circuit is comprised of a 1.5 pF series capacitor and a 2.8 pF shunt capacitor.

For the first case, a 2.8 pF shunt capacitor is employed, whereas in the second case this capacitor is replaced with a $\lambda/8$ open-circuited stub so that second harmonic signals can be adequately suppressed without affecting the fundamental-frequency input matching. This open stub has a sufficient bandwidth to suppress the second harmonics of both lower and upper fundamental frequencies, and as a result lower and upper IM3 are expected to have same amplitudes. Simulated results of IM3 levels for both cases are depicted in Fig. 2 wherein an improvement of around 2-3 dB is observed when the second harmonic trap is used.

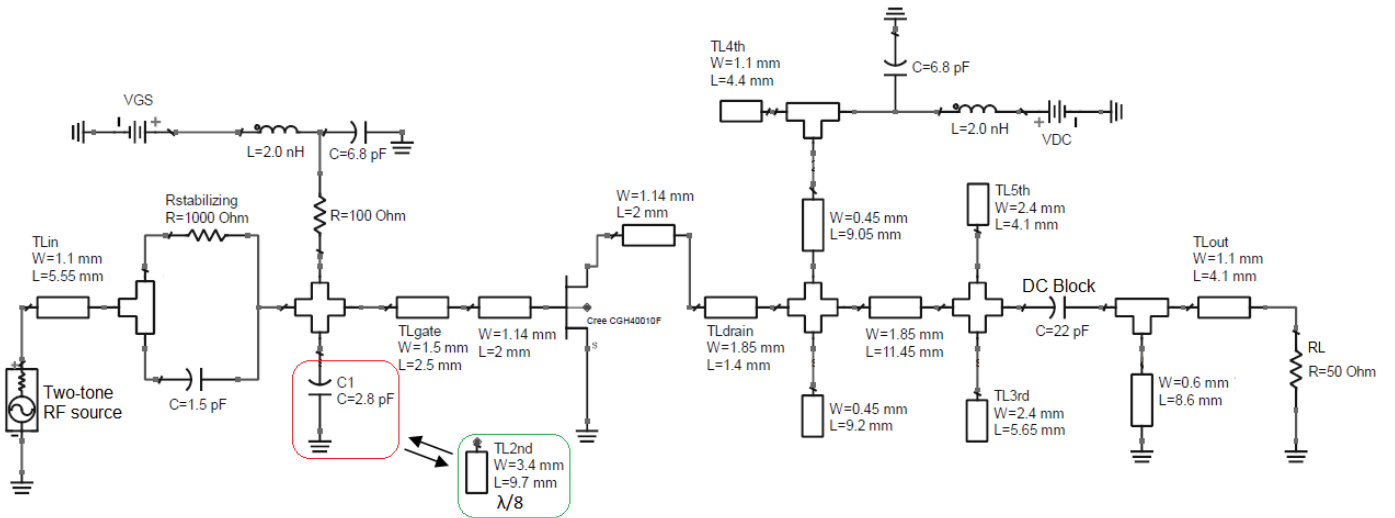


Fig. 1 Circuit schematic of the third-harmonic-peaking Class-EF PA. The first case uses C_1 whereas the second case uses an open-circuited stub TL2nd in place of C_1 .

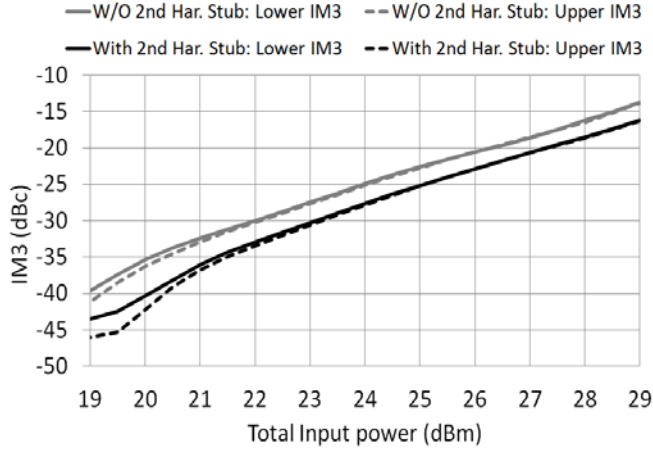


Fig. 2 Simulated third-order intermodulation product as a function of total input power.

Fig. 3 illustrates the output power levels for the two fundamental tones and the IM3 levels at the saturated point. It is observed that when the second-harmonic stub is used, IM3 levels are reduced by around 2.5 dB while the output power levels of each fundamental tone can be maintained at 37 dBm.

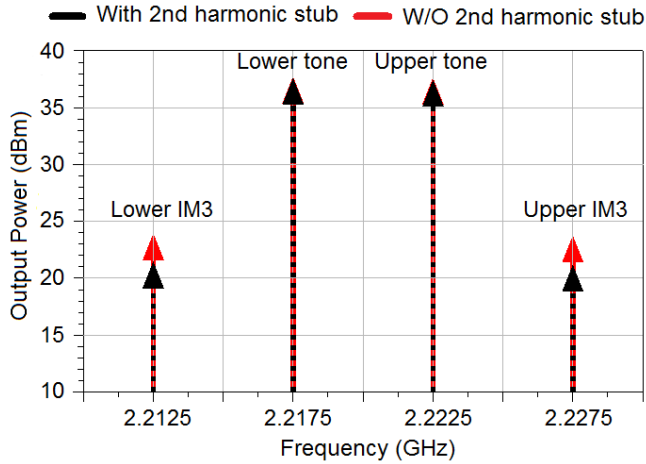


Fig. 3 Simulated output spectrum at saturated power, i.e., $P_{in} = 29$ dBm. The lower IM3 is reduced from 23.5 to 21 dBm and the upper IM3 is reduced from 23.3 to 20.7 dBm.

IV. CONCLUSION

The analysis of the intermodulation secondary mechanism due to the presence of a second-order harmonic component was presented in this paper. A second harmonic component is added at the amplifier input along with a two-tone signal. The mathematical derivation has shown the occurrence of extra generated in-band and out-of-band distortion terms above those which occur under the more ideal case of pure two-tone signal excitation. If a lower tone's second harmonic component is presented at the input, then an additional lower IM3 term is generated, and vice versa. A simulated example of a 2.22 GHz Class-EF amplifier is introduced to examine the effect of suppressing the second harmonics at the input on the IM3 levels. Results have shown a reduction in the IM3 levels by 2-3 dB when a harmonic termination stub is used.

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